FORECASTING AMERICAN STOCK OPTION PRICES[1]

Sangwoo Heo, University of Southern Indiana
Choon-Shan Lai, University of Southern Indiana

ABSTRACT
This study evaluates the performance of the MacMillan (1986), Barone-Adesi and Whaley (1987) (MBAW) model relative to the Black-Scholes (B-S) model in pricing American put options. We also investigate the implication of different choices of volatility to the predictive accuracy. Sets of model prices are generated using fifteen measures of volatility. The model-generated prices are compared to actual prices.

INTRODUCTION

Forecasting accuracy of option pricing models has always been a topic of interests. The forecasting ability of an option pricing model depends on the specification of the model and values of parameters. Unlike other parameters, volatility is not observed, therefore rendering a good estimator of volatility an essential factor of the predictive power of an option pricing model. While much has been focused on modifications and developments of models to improve forecasts, very little has been done on how the choice of a proxy for volatility affects the accuracy. This study tests the robustness of the MacMillan (1986), Barone-Adesi and Whaley (1987) (MBAW) model to fifteen measures of implied and historical volatility.

Theoretically, option contracts with the same underlying asset should be priced using the same volatility because the volatility of interests is that of the underlying asset return. However, previous studies show otherwise. Evidence on volatility smiles or skews is abundant. For example, deep in-the-money and out-of-the-money option contracts often have larger implied volatility than at-the-money contracts. Macbeth and Merville (1979) find that the implied variance imputed from the Black-Scholes(B-S) model is related to the moneyness and the time to expiration of the option in question.

In this study, we intend to evaluate the performance of the MacMillan (1986), Barone-Adesi and Whaley (1987) (MBAW) model in pricing American stock options. In the meantime, we investigate the impact of different choices of volatility measures on pricing accuracy. Sets of model prices of American put options are generated using fifteen different measures of volatility for both the MBAW and B-S specifications. The model-generated prices are compared with actual market prices.

This study is structured as follows: Section 1 consists of a literature survey. Section 2 describes the data and methodology. Section 3 reports the result and section 4 concludes the study and lays out future research.

LITERATURE SURVEY

A. Empirical Studies of American Option Pricing and Early Exercise Premium

Blomeyer and Johnson (1988) compare the ex post performance of the Geske and Johnson (GJ) American put valuation model with the Black-Scholes (BS) European put valuation model using transaction data of four stock option contracts from June through August 1978. Parkinson (1980) extreme value method is used to calculate the stock return standard deviation. Stock price range data in the 20 weeks preceding the week of the option transaction are used to calculate the standard deviation. Both undervalue market prices of put although the GJ model is significantly closer to market prices than the BS model. In addition, they find that GJ model capture a larger portion of the pricing bias of the BS model for in-the-money puts than out-of-the-money puts. One explanation of this is that the GJ model captures the early exercise premium that is argued to be more prevalent in in-the-money than out-of-the-money puts.

Zivney (1991) estimates the value of early exercise from deviation of an observed American put-call parity from an otherwise identical European put-call parity using transaction data of Standard & Poor (S&P) 100 index option contracts. He finds that the value of early exercise of calls increases with time to expiration, the risk-free rate of interest, the stock price and decreases with the exercise price. He also finds that the value of early exercise for put options is greater than call options.

Rhim and Kim (1999) report that MacMillan (1986), Barone-Adesi and Whaley (1987) (MBAW) overprice (underprice) in-the-money (out-of-the-money) puts. By decomposing the mispricing into pricing errors of the B-S model and the early premium, they find that the B-S model overprice (underprice) in-the-money (out-of-the-money) puts. Also, they find that the MBAW model tends to overprice (underprice) in-the-money (out-of-the-money) puts. The implied volatility estimate is obtained in the manner of Whaley (1982) by fitting the MBAW model to data of at-the-money options and then by minimizing the sum of squared errors between the observed price and model price. The early premium increases with the extent an option is in-the-money, the time to expiration, the interest rates and the volatility.

MacBeth and Merville (1979) find that the implied variance from the BS model is a function of time to expiration and moneyness of options. They report that out-of-the-money calls have smaller implied variances than in-the-money calls. They also report that out-of-the-money calls with shorter time to maturity to expiration have smaller implied variance than they longer-term counterparts. These results imply that there is a value of early exercise especially for in-the-money calls and longer term options.

B. Measures of Volatility

Macbeth and Merville (1979) focus on the implication of the choice of variance estimate. Because the variance is of the underlying asset, it should not vary across different option contracts with the same underlying asset. However, imputed variances from option pricing models show that the variance varies with exercise price and time-to-expiration. Macbeth and Merville (1979) show that implied variances of call options decrease with the strike price and increase with the time-to-maturity. If a constant variance is used in an option pricing model, mispricing may result due to varying nature of the variance. As a result, out-of-the-money calls are overpriced while in-the-money calls are underpriced by the BS model using a given estimated variance. In addition, the extent of mispricing decreases with the time left to expiration. Assuming that the "true" variance is the implied variance imputed from the B-S model for at-the-money call options with at least ninety days to expiration, Macbeth and Merville (1979) find that the B-S model overprices out-of-the-money calls and under-prices in-the-money calls. Furthermore, the pricing bias increases with the strike price and decreases with the time-to-expiration. Contradicting empirical results of mispricing of option pricing models may very well be explained away by different choices of variance estimator or different sample period of different volatility of underlying assets.

Later evidence on volatility smiles where the imputed variance is the largest with deep-in-the-money and deep-out-the-money options with very short time left to expiration mandate further look at the role of variance estimator in the pricing bias of option pricing models.

Ederington and Guan (2002) report that existing popular estimated variances used by practitioners and academicians on weekly data from January 1988 to April 1998 are upward biased measures of estimated volatility. Interestingly, they also find that these estimated variances overestimate the realized variances by a much larger margin for in-the-money calls (out-of-the-money puts) than out-of-the-money calls(in-the-money puts)

DATA AND METHODOLOGY

Daily data of American put option of Dow Jones Index (DJX) from November, 2000 to August, 2004 are used. All data are obtained from www.ivolatility.com .

We calibrate the MBAW as well as the B-S model with fifteen measures of implied and historical volatility: 1) the implied volatility over the past one day (iv1) as well as one(iv30), two(iv60), three(iv90), four(iv120), five(iv150) and six months(iv180) ; 2) the historical volatility over the past ten (hv10) and twenty days (hv20) as well as one (hv30), two (hv60), three (hv90), four(hv120), five(hv150) and six months(hv180).

After filtering for missing values and deleting the first entry of each new put options so that one-day implied volatility is available, we start out with 118771 observations. We further filter the data for the following criteria:

1. Options whose prices are less than their intrinsic values are omitted. (i.e. Price of the put option is less than the difference between the strike price and the spot price).
2. After calibrating the MBAW model with the full set of data, any observations with the size of pricing biases more than three standard deviations are omitted.

After filtering through the above two criteria, we are left with 108521 observations. We then calibrate both the MBAW and the B-S models.

To evaluate the accuracy of each model, we employ three measures of accuracy : bias, absolute bias and mean absolute percentage error (MAPE). The formulas are as follows:

$$Bias = P_{actual} - P_{model}$$

$$AbsoluteBias = | P_{actual} - P_{model} |$$

$$MAPE = \left(\frac{\sum_i^N \left| P_{actual,i} - P_{model,i} \right| / P_{actual,i} * 100\%}{N} \right)$$

where P_{actual} is the actual option price, P_{model} is the model-generated price and N is the number of observations.

RESULTS

As shown in Table 1, using all 108521 observations, the MBAW model outperforms the B-S model in most measures of volatility. Implied volatility overall predicts better than historical volatility. Implied volatility of the past day (iv1) outperforms all other measures of volatility by a large margin with MAPE around four percent.

When divided into groups according to moneyness as in Table 2-4, both models predict the best for in-the-money puts and the worst for out-of-the-money puts. One-day implied volatility (iv1) provides the most predictive power. Using one-day implied volatility, the MBAW model predicts better than the B-S model in all cases.

For in-the-money puts, both the B-S and MBAW models overvalue put options in most cases except when iv1, hv10, hv20 and hv30 are used in the B-S specification. Surprisingly, the B-S model has greater predictive accuracy than the MBAW model with most of the fifteen measures of volatility evaluated by all three measures of errors. Again, implied volatility generally performs better than historical volatility. Similar to the earlier finding with all observations, the one-day implied volatility (iv1) provides superior predictive power over other measures.

The general result of in-the-money puts holds for at-the-money options except that the MBAW model outperforms the B-S in most cases. In contrast, pricing errors of out-of-the-money puts are much larger than those for in-the-money and at-the-money puts in both specifications. The MBAW performs better than the B-S with all measures of volatility with out-of-the-money puts and with most measures of volatility with at-the-money puts. Similar to findings in the previous paragraphs, implied volatility generally performs better than historical volatility while one-day implied volatility (iv1) provides superior predictive power over other measures.

Observing the results for in-the-money and at-the-money options in Table 2-4, the B-S model performs better than the MBAW when both specifications overprices put options. Because the MBAW model, by construction of the model, always has the B-S price as the lower bound. As a result, the MBAW specification loses its charm when B-S over-estimates the price. However, when the B-S underestimates the price, the early exercise premium plays an important role. Among 108521 observations, B-S under estimates 60067 cases and MBAW performance is presented Table 5-8. It is worth exploring whether the discrepancy in pricing accuracy is due to the lack of adjustment for where the price level of the B-S is relative to the actual price.

As one-day implied volatility (iv1) offers the greatest accuracy in almost all cases. It is employed in both models to divide our sample into subgroups according to whether the B-S model overprices or under-prices options. The results are generally similar to the previously reported with some slight improvement of the predictive accuracy of the MBAW over the B-S.

Table 9-10 shows the differences between both model prices when the B-S model overprices options as well as under-prices options. The extent to which the MBAW price exceeds the B-S price is larger for observations in which the B-S price under-prices the actual price.

CONCLUSIONS AND FUTURE RESEARCH

The following is the summary of the results:

1. Using all observations in the sample, both the B-S and MBAW models under-price put options prices except when one-day implied volatility (iv1) is used in the MBAW model.
2. Using all observations in the sample, estimations using implied variances perform better than those using historical variances. The longer the data used for imputation, the more accurate the pricing is using historical volatility.
3. That said, however, one-day implied volatility (iv1) produces greater predictive accuracy than other measures of volatility by a large margin.
4. Using all observation in the sample**,** the MBAW model predicts better than the BS model with most measures of volatility.
5. The BS model performs better for in-the-money puts and the MBAW for at-the-money and out-of-the-money puts.

In sum, we have carried out a comprehensive analysis of performance of the MBAW relative to the B-S model in light of different choices of volatility estimators. We plan to explore the role of volatility estimators in predictive accuracy of option model pricing in more detail in the near future.

REFERENCES

Barone-Adesi, G. and Whaley, R., 1987, "Efficient Analytic Approximation of American Option Values", Journal of Finance 42, 301-320.

Black, F., and Scholes, M.S.,1973, "The Pricing of Options and Corporate Liabilities", Journal of Political Economy 81, 637-659.

Blomeyer, E. C., and H.Johnson, 1988, "An Empirical Examination of the Pricing of American Put Options", Journal of Financial and Quantitative Analysis, 13-22.

Brennan, M., and E. S. Schwartz, 1978, "Finite Difference Methods and Jump Processes Arising in the Pricing of Contingent Claims: A

Synthesis", Journal of Financial and Quantitative Analysis 13, 462-474.
Cakici, N., Chatterjee, S., and Wolf, A., 1993, "Empirical Tests of Valuation Models for Options on T-Note and T-Bond Futures", Journal of Futures Markets 13, 1-13.
Carr, P.,Jarrow, R, and Myneni, R.,1992, " Alternative Characterization of American Put Iptions", Mathematical Finance 2, 87-106.
Chen, R. and Yeh, S., 2002, " Analytical Upper Bounds for American Options Prices" Journal of Financial and Quantitative Analysis 37, 117-135.
Cox,J. C., S.A. Ross, and Rubinstein, N., 1979, " Option Pricing: A Simplified Approach", Journal of Financial Economics 3, 229-263.
De Roon, Frans., and Veld, Chris, 1996, "Put-Call Parities and the Value of Early Exercise for Put Options on a Performance Index", Journal of Futures Markets 16,71-80.
Ederington, L.H.,and Guan, Wei, 2002, "Measuring Implied Volatility: Is an Average Better? Which Average?" Journal of Futures Markets 22 811-837.
Geske, R., and H.E.Johnson, 1984, " The American Put Valued Analytically", Journal of Finance 39, 1511-1524.
Harvey, C.R. and R.E. Whaley, 1992, "Market Volatility Prediction and the Efficiency of the S&P 100 Index Option Market", Journal of Financial Economics 31, 43-73.
Hull, J., 1989, Options Futures and Other Derivative Securities, Englewood Cliffs, New Jersey: Prentice Hall
Jorian,P. and N.M. Stoughton, 1989, "An Empirical Investigation of the Early Exercise Premium of Foreign Currency Options", Journal of Futures Market 9, 365-375.
Kim, I.J., 1990, "The Analytic Valuation of American Options", Review of Financial Studies 3, 547-572.
Klemkosky, R.C., and B.G. Resnick, 1979, "Put-Call Parity and Market Efficiency", Journal of Finance 34, 1141-1155.
Longstaff, F., and Schwartz, E., 2001, "Valuing American Options by Simulation: A Simple Least-Squares Approach", Review of Financial Studies 14, 113-147.
Loudon, G.F., 1990, "American Put Pricing: Australian Evidence", Journal of Business, Finance and Accounting 17, 297-321.
MacBeth. J.D. and Merville, L.J., 1979, "An Empirical Examination of the Black-Scholes Call Option Pricing Model", Journal of Finance 34, 1173-1186.
MacMillan, W., 1986, " Analytical Approximation for American Put Options", Advances in Options and Futures Research 1, 119-139.
Overdahl, J.A., 1988, " The Early Exercise of Options on Treasury Bond Futures", Journal of Financial and Quantitative Analysis 23, 437-449.
Shastri, K.and Tandon, K., 1986, "An Empirical Test of A Valuation Model for American Options on Futures Contracts", Journal of Financial And Quantitative Analysis 21, 377-392.
Shaw, W., 1998, Modeling Financial Derivatives with Mathemetica, Cambridge: United Kingdom, Cambridge University Press.
Stampfli, J. and Goodman, V., 2001, The Mathematics of Finance: Modeling and Hedging, The Brooks/Cole series in advanced mathematics.
Whaley, R.E., 1982, " Valuation of American Call Options on Dividend-Paying Stocks", Journal of Financial Economics 10, 491-512.
Whaley, R.E., 1986, "Valuation of American Futures Options: Theory and Empirical Tests", Journal of Finance 41, 127-150
Zivney, T.L., 1991, " The Value of Early Exercise in Option Prices: An Empirical Investigation", Journal of financial and Quantitative Analysis 26, 129-138.

Table 1: All

Number of observation 108521

	P - Pe		ABS(P-Pe)		MPAE	
	mean	standard deviation	Mean	standard deviation	mean	standard deviation
BS iv1	0.037	0.2209	0.1176	0.1906	3.9930	5.4119
iv30	0.194	0.9292	0.6008	0.7348	31.2250	44.9410
iv60	0.1366	0.8352	0.5335	0.6569	29.2230	46.3090
iv90	0.1326	0.7639	0.4962	0.5957	28.4460	46.2150
iv120	0.1439	0.7178	0.4777	0.5548	28.2320	46.2110
iv150	0.1582	0.6846	0.4663	0.5256	28.1970	46.2060
iv180	0.1682	0.6628	0.4599	0.5060	28.1740	46.0020
hv10	0.5189	1.4798	1.0137	1.1965	42.5430	46.7880
hv20	0.4845	1.3817	0.9514	1.1130	40.8950	43.6130
hv30	0.4575	1.2713	0.9042	1.0040	39.2900	41.2430
hv60	0.4034	1.0874	0.7874	0.8516	36.0530	39.3950
hv90	0.3723	0.9918	0.7067	0.7892	34.4040	40.1440
hv120	0.3533	0.9132	0.6566	0.7264	33.1500	39.2440
hv150	0.3381	0.8826	0.6383	0.6970	32.4730	40.2030
hv180	0.3238	0.8888	0.6413	0.6954	32.1150	41.3790
Mbaw iv1	-0.011	0.2057	0.1155	0.1706	3.9503	5.3443
iv30	0.15	0.9739	0.6131	0.7715	31.1940	44.8650
iv60	0.0926	0.8840	0.5509	0.6975	29.2120	46.2170
iv90	0.0886	0.8130	0.5151	0.6352	28.4310	46.1150
iv120	0.0998	0.7649	0.4959	0.5908	28.2010	46.1110
iv150	0.114	0.7294	0.4839	0.5576	28.1530	46.1050
iv180	0.1239	0.7058	0.477	0.5348	28.1210	45.9010
hv10	0.4722	1.5035	1.0028	1.2156	42.4080	46.8500
hv20	0.4385	1.4099	0.9426	1.1365	40.7690	43.6610
hv30	0.4117	1.3012	0.8984	1.0274	39.1850	41.2610
hv60	0.3581	1.1232	0.7885	0.8764	35.9760	39.3540
hv90	0.3273	1.0271	0.712	0.8093	34.3230	40.0840
hv120	0.3083	0.9457	0.6631	0.7414	33.0610	39.1700
hv150	0.2927	0.9085	0.6429	0.7055	32.3640	40.1300
hv180	0.278	0.9064	0.6422	0.6974	31.9760	41.3150

Table 2: In-the-money

Number of observation 40699 .

	P - Pe		ABS(P-Pe)		MPAE	
	mean	standard deviation	mean	standard deviation	mean	standard deviation
BS iv1	0.0654	0.31202	0.182	0.26173	1.601	2.1794
iv30	-0.295	0.97332	0.5442	0.85937	4.1237	5.1604
iv60	-0.351	0.8626	0.4987	0.78627	3.788	4.4206
iv90	-0.344	0.76865	0.4594	0.70583	3.5602	3.8657
iv120	-0.323	0.70136	0.4312	0.64047	3.4157	3.5111
iv150	-0.3	0.64968	0.4081	0.5877	3.3123	3.2736
iv180	-0.282	0.61506	0.3938	0.55023	3.2819	3.1767
hv10	0.0829	1.6288	0.897	1.3621	7.6277	10.079
hv20	0.0544	1.508	0.8232	1.2646	6.9791	9.0769
hv30	0.0329	1.3655	0.7927	1.1123	6.7278	8.1213
hv60	-0.026	1.1442	0.7102	0.89741	5.9664	6.4767
hv90	-0.056	1.0271	0.616	0.82379	5.2832	5.9639
hv120	-0.074	0.9144	0.5576	0.72848	4.8724	5.3898
hv150	-0.086	0.86601	0.5523	0.6726	4.8899	5.2227
hv180	-0.093	0.86859	0.5605	0.67008	5.0563	5.4116
Mbaw iv1	-0.03	0.2891	0.1787	0.22926	1.5776	2.0332
iv30	-0.384	1.0426	0.5839	0.94532	4.3484	5.5619
iv60	-0.439	0.94051	0.5537	0.87777	4.1141	4.8757
iv90	-0.433	0.84883	0.5209	0.79768	3.9275	4.3162
iv120	-0.412	0.77935	0.4931	0.73057	3.7791	3.9262
iv150	-0.389	0.72497	0.4703	0.67503	3.6751	3.6513
iv180	-0.372	0.68799	0.4563	0.63495	3.6456	3.5169
hv10	-0.016	1.6547	0.875	1.4045	7.4601	10.208
hv20	-0.042	1.5429	0.8068	1.3158	6.832	9.2467
hv30	-0.063	1.4039	0.7822	1.1675	6.6171	8.2922
hv60	-0.119	1.1958	0.7162	0.96503	5.9567	6.6852
hv90	-0.148	1.0794	0.6375	0.88351	5.3796	6.1346
hv120	-0.166	0.96294	0.5865	0.78152	5.0223	5.5325
hv150	-0.18	0.90078	0.5781	0.7139	5.0183	5.299
hv180	-0.189	0.88572	0.5794	0.69593	5.1458	5.4119

Table 3: At-the-money

Number of observation 12226

	P - Pe		ABS(P-Pe)		MPAE	
	mean	standard deviation	mean	standard deviation	mean	standard deviation
BS iv1	0.0222	0.19151	0.1205	0.15048	3.8693	4.6168
iv30	0.1249	0.68001	0.4191	0.54988	9.4133	8.3317
iv60	0.0254	0.52975	0.3142	0.42724	7.7359	7.8765
iv90	-0.004	0.43448	0.2697	0.34066	7.565	8.4081
iv120	-0.006	0.38472	0.2553	0.28784	7.8244	8.8987
iv150	-0.003	0.35342	0.2485	0.25131	8.1301	9.3424
iv180	-0.002	0.33946	0.2487	0.23107	8.5122	9.7906
hv10	0.6136	1.4138	1.0475	1.1305	25.388	17.713
hv20	0.5557	1.3024	0.9765	1.0254	23.596	15.212
hv30	0.5171	1.1975	0.9286	0.91592	22.365	13.841
hv60	0.4305	1.0077	0.7952	0.75396	19.829	12.858
hv90	0.3714	0.90165	0.6916	0.68746	17.879	12.356
hv120	0.3255	0.82119	0.6234	0.62582	16.388	11.838
hv150	0.2902	0.80141	0.5977	0.60766	15.902	12.299
hv180	0.2511	0.81712	0.5986	0.6102	16.22	12.724
Mbaw iv1	-0.005	0.1802	0.1174	0.13678	3.8209	4.5624
iv30	0.0976	0.6894	0.4175	0.55724	9.3795	8.3662
iv60	-0.002	0.53966	0.3136	0.43919	7.7181	7.9369
iv90	-0.031	0.4409	0.2677	0.35172	7.5251	8.4527
iv120	-0.034	0.38487	0.2508	0.29392	7.7395	8.9228
iv150	-0.03	0.34749	0.2421	0.25106	8.021	9.3539
iv180	-0.029	0.32944	0.2416	0.22579	8.3935	9.7958
hv10	0.5872	1.4176	1.0396	1.1285	25.262	17.723
hv20	0.5291	1.3114	0.9682	1.0306	23.456	15.285
hv30	0.4904	1.2063	0.9228	0.9188	22.257	13.876
hv60	0.4039	1.018	0.7944	0.75386	19.797	12.855
hv90	0.3448	0.90968	0.6931	0.68264	17.895	12.294
hv120	0.2989	0.82497	0.6241	0.61679	16.413	11.749
hv150	0.2636	0.79902	0.5955	0.59434	15.893	12.203
hv180	0.2245	0.80881	0.5936	0.59343	16.171	12.627

Table 4: Out of the Money

Number of observation 55596

	P - Pe		ABS(P-Pe)		MPAE	
	mean	standard deviation	mean	standard deviation	mean	standard deviation
BS iv1	0.0194	0.12232	0.0699	0.10226	5.7712	6.4539
iv30	0.5674	0.75775	0.6823	0.65625	55.861	51.56
iv60	0.5177	0.6583	0.6073	0.57673	52.567	55.118
iv90	0.5117	0.5922	0.573	0.53308	51.255	55.435
iv120	0.5186	0.55518	0.5606	0.5128	50.886	55.555
iv150	0.5288	0.53096	0.5569	0.50143	50.826	55.556
iv180	0.5351	0.51565	0.5547	0.49444	50.719	55.275
hv10	0.8173	1.2895	1.0917	1.0673	71.876	48.085
hv20	0.7838	1.2094	1.0397	0.99804	69.528	43.288
hv30	0.7553	1.1194	0.9804	0.92864	66.849	40.388
hv60	0.7116	0.94678	0.8421	0.83284	61.645	39.834
hv90	0.6859	0.85824	0.7765	0.77729	59.356	42.221
hv120	0.6719	0.7951	0.7363	0.73584	57.536	41.379
hv150	0.6594	0.77112	0.7102	0.72461	56.309	43.774
hv180	0.6451	0.78183	0.7098	0.72361	55.419	46.373
Mbaw iv1	0.001	0.11761	0.0687	0.09545	5.7157	6.3958
iv30	0.5525	0.76149	0.6774	0.65287	55.643	51.585
iv60	0.5025	0.66115	0.601	0.57304	52.312	55.153
iv90	0.4965	0.59296	0.5652	0.52781	50.966	55.482
iv120	0.5035	0.55342	0.5518	0.50534	50.578	55.607
iv150	0.5139	0.52692	0.547	0.4924	50.5	55.616
iv180	0.5202	0.5101	0.5439	0.48478	50.377	55.345
hv10	0.8044	1.2953	1.0883	1.068	71.762	48.151
hv20	0.7704	1.2159	1.0364	0.99886	69.419	43.326
hv30	0.7416	1.1258	0.9781	0.92773	66.749	40.389
hv60	0.6973	0.95174	0.8402	0.82828	61.51	39.81
hv90	0.6715	0.86043	0.7708	0.77273	59.123	42.283
hv120	0.6575	0.79417	0.7277	0.73035	57.248	41.48
hv150	0.6452	0.76715	0.7008	0.71674	56.005	43.866
hv180	0.6312	0.77519	0.6989	0.71474	55.093	46.466

Table 5: The B-S price < The Actual Price, All

Number of observation 60067

	P - Pe		ABS(P-Pe)		MPAE	
	mean	standard deviation	mean	standard deviation	mean	standard deviation
BS iv1	0.1397	0.22999	0.1397	0.22999	4.2339	5.5461
iv30	0.233	0.96021	0.6318	0.7597	32.326	46.679
iv60	0.1782	0.8641	0.5574	0.68389	30.263	48.62
iv90	0.1784	0.78841	0.5171	0.62131	29.469	48.717
iv120	0.1951	0.73579	0.4969	0.57664	29.252	48.651
iv150	0.2143	0.69512	0.4835	0.54348	29.197	48.587
iv180	0.2291	0.66709	0.4754	0.52103	29.167	48.36
hv10	0.4973	1.5758	1.0681	1.2608	42.951	48.64
hv20	0.4723	1.4401	0.993	1.1449	41.249	44.689
hv30	0.4382	1.3266	0.9447	1.0293	39.634	42.068
hv60	0.3581	1.157	0.8296	0.88248	36.221	40.101
hv90	0.3416	1.0535	0.7423	0.82195	34.608	41.06
hv120	0.3413	0.9485	0.6807	0.74346	33.286	40.18
hv150	0.3662	0.88523	0.6522	0.70166	32.792	41.471
hv180	0.3749	0.87776	0.6521	0.69692	32.536	42.912
Mbaw iv1	0.0774	0.18199	0.1105	0.16398	3.8149	5.3715
iv30	0.1773	1.0162	0.6458	0.80436	32.269	46.594
iv60	0.1225	0.92508	0.5777	0.73281	30.232	48.518
iv90	0.1227	0.84987	0.539	0.66847	29.432	48.607
iv120	0.1392	0.79448	0.5174	0.6188	29.191	48.542
iv150	0.1583	0.75107	0.5028	0.57995	29.119	48.478
iv180	0.173	0.7208	0.4939	0.55277	29.075	48.252
hv10	0.4376	1.6077	1.0554	1.2892	42.783	48.714
hv20	0.4136	1.4764	0.9828	1.1768	41.092	44.747
hv30	0.3798	1.3642	0.9389	1.0601	39.511	42.088
hv60	0.3004	1.2048	0.8337	0.92019	36.15	40.05
hv90	0.2842	1.1009	0.7517	0.85302	34.534	40.981
hv120	0.2841	0.99355	0.6917	0.76775	33.203	40.085
hv150	0.3086	0.9205	0.6588	0.71311	32.665	41.383
hv180	0.3168	0.9022	0.6527	0.69874	32.36	42.841

Table 6: The B-S price <The Actual Price, in-the-money

Number of observation 22891

	P - Pe		ABS(P-Pe)		MPAE	
	mean	standard deviation	mean	standard deviation	mean	standard deviation
BS iv1	0.22	0.31454	0.22	0.31454	1.7953	2.37
iv30	-0.257	1.0366	0.5628	0.90751	3.9603	5.1407
iv60	-0.305	0.92555	0.5012	0.83588	3.4732	4.4164
iv90	-0.291	0.82761	0.4526	0.75157	3.1816	3.8121
iv120	-0.262	0.75126	0.4182	0.67666	3.0131	3.384
iv150	-0.231	0.68832	0.3891	0.61284	2.8874	3.0706
iv180	-0.206	0.64341	0.3697	0.56533	2.8366	2.9094
hv10	0.0377	1.7742	0.9848	1.4762	7.9474	10.286
hv20	0.0282	1.5957	0.8871	1.3267	7.2179	9.1362
hv30	-1E-03	1.4489	0.8634	1.1635	7.0422	8.2423
hv60	-0.091	1.2561	0.8012	0.97159	6.3672	6.6698
hv90	-0.097	1.133	0.6936	0.90106	5.6165	6.1923
hv120	-0.084	0.98494	0.6097	0.77816	5.0748	5.5134
hv150	-0.042	0.88419	0.565	0.68144	4.8516	5.2168
hv180	-0.017	0.86247	0.5555	0.65994	4.8877	5.3301
Mbaw iv1	0.0966	0.25493	0.1673	0.21528	1.4473	1.8644
iv30	-0.368	1.1198	0.6096	1.0088	4.2158	5.6183
iv60	-0.416	1.0185	0.5668	0.94295	3.856	4.969
iv90	-0.402	0.92306	0.5253	0.85891	3.6079	4.3676
iv120	-0.373	0.84368	0.4899	0.78161	3.4199	3.8936
iv150	-0.343	0.77781	0.4605	0.71434	3.2876	3.5335
iv180	-0.318	0.73035	0.441	0.66343	3.2342	3.3231
hv10	-0.087	1.8094	0.9599	1.5362	7.7396	10.464
hv20	-0.093	1.6401	0.8686	1.3943	7.0329	9.3637
hv30	-0.121	1.4959	0.8537	1.2343	6.9127	8.4732
hv60	-0.208	1.3226	0.813	1.0637	6.3719	6.9848
hv90	-0.212	1.2004	0.7243	0.98046	5.765	6.4456
hv120	-0.2	1.0507	0.6502	0.84915	5.2977	5.741
hv150	-0.159	0.9325	0.5986	0.73252	5.031	5.3241
hv180	-0.136	0.88971	0.5781	0.68993	5.0046	5.3288

Table 7: The B-S price <The Actual Price, at-the-money

Number of observation 6273

	P - Pe		ABS(P-Pe)		MPAE	
	mean	standard deviation	mean	standard deviation	mean	standard deviation
BS iv1	0.139	0.1792	0.139	0.1792	3.8397	4.3213
iv30	0.2091	0.71633	0.4675	0.58164	9.915	8.2248
iv60	0.1045	0.56427	0.3399	0.46237	7.4576	7.0545
iv90	0.0783	0.46706	0.2885	0.37551	7.0011	7.1178
iv120	0.0804	0.41448	0.2739	0.32129	7.1955	7.4042
iv150	0.0877	0.37781	0.2648	0.28337	7.4105	7.7504
iv180	0.0924	0.3584	0.2618	0.26161	7.6877	8.1628
hv10	0.6538	1.5288	1.1461	1.2046	26.085	17.474
hv20	0.5961	1.3765	1.065	1.0562	24.452	14.924
hv30	0.5487	1.2811	1.0166	0.95322	23.281	13.821
hv60	0.4287	1.0997	0.8755	0.79158	20.509	12.766
hv90	0.3753	0.97992	0.7533	0.73046	18.296	12.486
hv120	0.3378	0.87289	0.6663	0.65725	16.471	11.947
hv150	0.3355	0.82588	0.6273	0.63329	15.698	12.228
hv180	0.315	0.83441	0.6223	0.6389	15.794	12.631
Mbaw iv1	0.1011	0.15314	0.1176	0.1409	3.5086	4.1566
iv30	0.1713	0.72967	0.4632	0.58922	9.8184	8.2526
iv60	0.0664	0.57892	0.3362	0.47592	7.3728	7.1215
iv90	0.0401	0.4774	0.2825	0.38693	6.879	7.1609
iv120	0.0423	0.41586	0.2633	0.32461	7.0013	7.4088
iv150	0.0497	0.37073	0.251	0.27731	7.1777	7.7329
iv180	0.0544	0.34552	0.2464	0.24826	7.4351	8.1397
hv10	0.6166	1.5354	1.1352	1.2037	25.909	17.486
hv20	0.5587	1.388	1.0528	1.0631	24.251	15.039
hv30	0.5113	1.2929	1.0088	0.95659	23.143	13.874
hv60	0.3911	1.118	0.8777	0.79519	20.507	12.755
hv90	0.3377	0.9961	0.7602	0.72684	18.37	12.367
hv120	0.3004	0.88413	0.6718	0.64856	16.556	11.802
hv150	0.2983	0.82633	0.6249	0.61741	15.69	12.098
hv180	0.2779	0.82564	0.6144	0.61756	15.71	12.506

Table 8: The B-S price <The Actual Price, out-of-the-money

Number of observation 30903

	P - Pe		ABS(P-Pe)		MPAE	
	mean	standard deviation	mean	standard deviation	mean	standard deviation
BS iv1	0.0803	0.12128	0.0803	0.12128	6.1203	6.6214
iv30	0.6005	0.76116	0.7162	0.65345	57.888	53.391
iv60	0.5513	0.66236	0.6432	0.57345	54.736	57.742
iv90	0.5465	0.59641	0.6113	0.52987	53.501	58.311
iv120	0.5567	0.55816	0.6005	0.51069	53.166	58.313
iv150	0.5696	0.53218	0.5978	0.50026	53.108	58.212
iv180	0.5789	0.51585	0.5971	0.49467	53.03	57.874
hv10	0.8061	1.3281	1.1139	1.0829	72.304	51.297
hv20	0.7761	1.2325	1.0568	1.0022	69.867	45.174
hv30	0.7411	1.1383	0.9903	0.92961	67.096	41.899
hv60	0.6764	0.96763	0.8412	0.82836	61.524	41.342
hv90	0.6593	0.87655	0.7761	0.77498	59.394	43.886
hv120	0.6572	0.80091	0.7363	0.7289	57.597	42.995
hv150	0.6748	0.76599	0.7219	0.72178	56.958	45.506
hv180	0.6774	0.77404	0.7298	0.72492	56.414	48.297
Mbaw iv1	0.0583	0.10356	0.0671	0.09811	5.6308	6.5217
iv30	0.5823	0.76545	0.7096	0.64922	57.606	53.437
iv60	0.5327	0.66583	0.6349	0.56924	54.41	57.802
iv90	0.528	0.59744	0.6012	0.5238	53.139	58.384
iv120	0.5384	0.55583	0.5893	0.50156	52.784	58.388
iv150	0.5514	0.52691	0.5853	0.48907	52.707	58.295
iv180	0.5609	0.50851	0.5833	0.48258	52.609	57.969
hv10	0.7899	1.3363	1.11	1.0851	72.165	51.368
hv20	0.7594	1.2407	1.0532	1.0034	69.74	45.204
hv30	0.7241	1.1458	0.9879	0.92807	66.98	41.891
hv60	0.6584	0.97522	0.8401	0.82385	61.382	41.296
hv90	0.641	0.88118	0.7702	0.77081	59.125	43.951
hv120	0.6392	0.80179	0.7265	0.72367	57.252	43.117
hv150	0.6573	0.76191	0.7103	0.71278	56.58	45.628
hv180	0.6603	0.76617	0.7159	0.71456	56.003	48.426

Table 9: Differences between the MBAW price and the B-S price and Differences between these two prices as a percentage of the option price (OP), the B-S price >= the actual Price

All

	mbaw-bs		(mbaw-bs)/OP	
	mean	standard deviation	mean	standard deviation
iv1	0.0312	0.1016	0.4240	0.9571
iv30	0.0295	0.0993	0.3248	0.8163
iv60	0.0295	0.0982	0.3289	0.8126
iv90	0.0296	0.0978	0.3295	0.8086
iv120	0.0296	0.0978	0.3293	0.8060
iv150	0.0296	0.0978	0.3289	0.8034
iv180	0.0296	0.0978	0.3293	0.8031
hv10	0.0307	0.1202	0.3125	0.9068
hv20	0.0304	0.1160	0.3144	0.8891
hv30	0.0303	0.1126	0.3161	0.8752
hv60	0.0297	0.1072	0.3173	0.8413
hv90	0.0297	0.1046	0.3194	0.8252
hv120	0.0299	0.1045	0.3198	0.8188
hv150	0.0305	0.1050	0.3248	0.8256
hv180	0.0308	0.1059	0.3264	0.8271

At

	mbaw-bs		(mbaw-bs)/OP	
	mean	standard deviation	mean	standard deviation
	AT			
iv1	0.0163	0.05501	0.2495	0.6684
iv30	0.016	0.05514	0.2439	0.6634
iv60	0.0161	0.0552	0.246	0.6647
iv90	0.0162	0.05507	0.2469	0.6646
iv120	0.0162	0.05495	0.2475	0.6645
iv150	0.0162	0.05479	0.2477	0.6638
iv180	0.0162	0.0547	0.2478	0.6636
hv10	0.015	0.05335	0.2347	0.6543
hv20	0.0153	0.05551	0.2374	0.6677
hv30	0.0153	0.05554	0.2377	0.6672
hv60	0.0151	0.05386	0.2345	0.6534
hv90	0.0151	0.0532	0.2342	0.6479
hv120	0.0152	0.05296	0.2347	0.6465
hv150	0.0154	0.05335	0.2376	0.6513
hv180	0.0155	0.05323	0.2391	0.6516

IN

	mbaw-bs		(mbaw-bs)/OP	
	mean	standard deviation	mean	standard deviation
iv1	0.0601	0.1534	0.3939	0.9776
iv30	0.0598	0.1500	0.3902	0.9618
iv60	0.0596	0.1480	0.3892	0.9510
iv90	0.0596	0.1475	0.3899	0.9491
iv120	0.0597	0.1475	0.3912	0.9502
iv150	0.0598	0.1476	0.3923	0.9514
iv180	0.0599	0.1477	0.3931	0.9523
hv10	0.0662	0.1867	0.4374	1.1852
hv20	0.0649	0.1791	0.4295	1.1423
hv30	0.0641	0.1729	0.4246	1.1114
hv60	0.0624	0.1640	0.4117	1.0520
hv90	0.0621	0.1597	0.4068	1.0208
hv120	0.0627	0.1596	0.4088	1.0182
hv150	0.0638	0.1603	0.4154	1.0256
hv180	0.0647	0.1619	0.4210	1.0386

OUT

	mbaw-bs		(mbaw-bs)/OP	
	mean	standard deviation	mean	standard deviation
iv1	0.014	0.03998	0.4878	0.9943
iv30	0.0109	0.03645	0.2971	0.7272
iv60	0.0111	0.03661	0.3054	0.7295
iv90	0.0111	0.03635	0.3059	0.7226
iv120	0.011	0.03609	0.3044	0.7156
iv150	0.011	0.03578	0.3027	0.7089
iv180	0.011	0.03567	0.3029	0.7072
hv10	0.0089	0.03291	0.2412	0.6924
hv20	0.0093	0.03446	0.25	0.6984
hv30	0.0095	0.0355	0.2567	0.7011
hv60	0.0097	0.0348	0.2692	0.6910
hv90	0.0098	0.03419	0.2769	0.6888
hv120	0.0098	0.03362	0.2762	0.6766
hv150	0.01	0.03382	0.2805	0.6833
hv180	0.01	0.03347	0.2793	0.6720

Table 10: Differences between the MBAW price and the B-S price and Differences between these two prices as a percentage of the option price (OP), the B-S price < the actual Price

ALL

	mbaw-bs		(mbaw-bs)/OP	
	mean	standard deviation	mean	standard deviation
iv1	0.0623	0.1730	0.7327	1.4356
iv30	0.0557	0.1520	0.5764	1.2206
iv60	0.0556	0.1503	0.5809	1.2141
iv90	0.0557	0.1503	0.5813	1.2109
iv120	0.0559	0.1511	0.5791	1.2090
iv150	0.056	0.1516	0.5775	1.2069
iv180	0.0561	0.1520	0.5769	1.2061
hv10	0.0598	0.1794	0.5795	1.3671
hv20	0.0586	0.1701	0.5798	1.3240
hv30	0.0584	0.1676	0.5864	1.3170
hv60	0.0577	0.1611	0.5964	1.2850
hv90	0.0574	0.1580	0.5939	1.2551
hv120	0.0571	0.1558	0.5893	1.2334
hv150	0.0576	0.1580	0.5858	1.2330
hv180	0.0581	0.1606	0.5832	1.2354

AT

	mbaw-bs		(mbaw-bs)/OP	
	mean	standard deviation	mean	standard deviation
iv1	0.0379	0.1023	0.5284	1.2305
iv30	0.0379	0.1028	0.5251	1.2298
iv60	0.0381	0.1030	0.5275	1.2314
iv90	0.0381	0.1029	0.5283	1.2308
iv120	0.0381	0.1027	0.5284	1.2291
iv150	0.038	0.1024	0.5279	1.2274
iv180	0.038	0.1023	0.5275	1.2266
hv10	0.0372	0.1022	0.5192	1.2294
hv20	0.0374	0.1028	0.5208	1.2335
hv30	0.0374	0.1028	0.521	1.2336
hv60	0.0376	0.1032	0.522	1.2346
hv90	0.0376	0.1029	0.5209	1.2312
hv120	0.0374	0.1024	0.5188	1.2276
hv150	0.0372	0.1017	0.518	1.2231
hv180	0.0372	0.1014	0.5181	1.2208

IN

	mbaw-bs		(mbaw-bs)/OP	
	mean	standard deviation	mean	standard deviation
iv1	0.1234	0.2564	0.7872	1.6470
iv30	0.1113	0.2219	0.7147	1.4657
iv60	0.1105	0.2189	0.7092	1.4464
iv90	0.1108	0.2190	0.7109	1.4474
iv120	0.1115	0.2204	0.7159	1.4591
iv150	0.112	0.2215	0.719	1.4653
iv180	0.1124	0.2222	0.7211	1.4687
hv10	0.1248	0.2673	0.8088	1.7620
hv20	0.1211	0.2517	0.7849	1.6642
hv30	0.1199	0.2476	0.7767	1.6347
hv60	0.1169	0.2367	0.7535	1.5515
hv90	0.1157	0.2315	0.7391	1.5069
hv120	0.1153	0.2279	0.7314	1.4768
hv150	0.1172	0.2317	0.7427	1.5005
hv180	0.1191	0.2361	0.7542	1.5280

OUT

	mbaw-bs		(mbaw-bs)/OP	
	mean	standard deviation	mean	standard deviation
iv1	0.022	0.0533	0.7338	1.2957
iv30	0.0182	0.0502	0.4843	0.9873
iv60	0.0185	0.0505	0.4967	0.9940
iv90	0.0185	0.0502	0.496	0.9852
iv120	0.0183	0.0496	0.4881	0.9670
iv150	0.0181	0.0490	0.4827	0.9545
iv180	0.018	0.0487	0.4801	0.9482
hv10	0.0162	0.0485	0.4218	0.9806
hv20	0.0167	0.0493	0.4399	0.9978
hv30	0.0171	0.0495	0.4587	1.0195
hv60	0.018	0.0506	0.4952	1.0429
hv90	0.0182	0.0502	0.5012	1.0233
hv120	0.018	0.0495	0.4983	1.0057
hv150	0.0175	0.0482	0.4832	0.9767
hv180	0.0171	0.0472	0.4698	0.9482