CVAR-Constrained Multi-Period Power Portfolio Optimization

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ABSTRACT

We consider power portfolio optimization of real and contractual assets, including derivative instruments in a multi-period setting. A model is introduced that incorporates fixed transmission rights in a three-node unidirectional network in order to evaluate the significance of transmission constraints. We use data from the PJM, which is located in the eastern United States for model implementation. The simulation results show that transmission constraints and fixed transmission rights can have a significant effect on the choices a utility will make when dealing with power procurement. Our results imply that companies should not only hedge the risk of unknown power prices but also unknown transmission congestion.

INTRODUCTION

The pre-deregulation electricity markets were characterized by predictable power prices and utility-owned power projects. In the 1990s, regulators in certain states began to explore deregulating the markets. They theorized that the increase in competition would result in lower power prices for consumers. By the late-90s, some states had deregulated their markets and others followed in the early 2000s. The result for utilities was the

introduction of competition for customers and more price volatility in power procurement. Companies that were interested in entering the power market could now apply to solicit customers and/or sell electricity on the wholesale market. All players in the electric market were faced with the risk of uncertain fuel prices, weather conditions and, therefore, unpredictable power prices due to electricity's non-storable nature. In order to combat the uncertainty, companies have developed hedging strategies. Historically, models have accounted for differing time horizons, types of generation and risk. Many have commented that transmission constraints should be considered, but none has attempted to incorporate them into the model. This paper builds on previously-developed models for the electric industry dealing with a multi-period portfolio optimization incorporating conditional value at risk (CVaR). Our model incorporates fixed transmission rights in a three-node unidirectional network in order to evaluate the significance of transmission considerations in power portfolio optimization.
The stochastic nonlinear mixed-integer model presented shows that transmission constraints and fixed transmission rights can have a significant effect on the choices a utility will make when dealing with power procurement. It is demonstrated that the inclusions drastically decrease the value of the objective function.

LITERATURE

There has been a vast amount of research done in recent years dealing with power portfolio optimization. The non-storable nature of electricity and the increasing complexity of financial instruments as a tool for hedging against risk make the area of research very useful in the real world. Work done previously in this area provides models to help energy companies optimize profits, but very few incorporate transmission constraints into their models. The contribution of this research is to help companies not only hedge the risk of unknown power prices but also unknown transmission congestion. Literature on power portfolio optimization has used different measurements of risk: value at risk (VaR), CVaR and variation of spot price. While VaR had historically been used as a risk measure for electricity markets, the advantages of CVaR include more robust mathematical properties for a more accurate measure of extreme risk situations contained in the tail of the distribution. Likewise, different articles have incorporated financial portfolios of varying scope. Some focus primarily on day ahead, forward, and spot prices. Other papers also include power purchase agreements and/or options. Some authors use generation location or performance (such as ramp rates, heat rates, etc.) as constraints while others focus on different execution and reservation prices for options. Varying timeframes are used in the models, too.

Kleindorfer and Wu (2003) gave an excellent review of financial instruments and how they can be used in power markets. It integrated contracting and market structure with operational decisions and explained in detail why a company would choose the forward/options market over the spot market and vice versa, using a graph with corresponding costs. Basically, the more “make to order” businesses would favor the contracts market due to variability in product. Kleindorfer and Li (2005) developed a model that decreases the allowable time period for using the VaR measure from one year to one month, which allows for a more realistic decision timeframe. A Monte Carlo simulation was run to arrive at different portfolio combinations. Xu et al (2006) focused on the issue facing a utility of how to best procure power for its customers. They used semi-variances of spot market transactions to measure risk and offer a model that can analyze the procurement situation with different types of power generation and financial tools. Oum et al (2006) also attempted to aid utilities in finding the optimal financial portfolios given a set amount of available resources for additional capacity taking into account fluctuating demand. It addressed the problem of developing an optimal hedging portfolio consisting of forward and options contracts for a risk-averse load-serving entity when price and volumetric risks are present and correlated. While the focus of Oum et al (2006) was optimizing available resources, the Murphy and Smeers (2005) studied capacity expansion. Utilities seek to optimize their generation portfolios in order to have a sufficient amount of baseload, peaking and cycling capacity while minimizing costs. Here fuel costs can still be passed along to the customers and an oligopolistic market (where each player can influence prices) is assumed. The paper was presented based on a two-stage model: in the first stage investment decisions are made and in the second stage operational decisions are made.

Kwon et al (2006) focused more on the agent that sells power either through long-term purchase agreements or through other financial arrangements. The model developed then aides the selling agent in developing the optimal mix of custom contracts. The authors used a two-stage stochastic programming model where the first stage's result is the quantity of forward contracts to buy, and the second stage gives the electric capacity to make or buy in future time periods. In a recent study, Conejo et al (2008) showed how a power producer can optimize its profits by utilizing forward contracts when they can be signed up to one year in advance. Only two tools were considered for power purchases in their model development: forward contracts and the "pool market" – day ahead, rather than real time. The decision of when to participate in the forward market was a complex one, involving lots of uncertainty over an extensive period of time.

A particularly interesting article was written by Olmos and Neuhoff (2006) and deals with finding a balancing point in a transmission network where companies cannot utilize market power by owning transmission rights. Although an actual logical point was not found in the European Union's network, the model is a good start to researching equitable fixed transmission rights .

MODEL AND DATA

The objective of our model is to determine the profit-maximizing combination of different power purchasing portfolios given transmission constraints and risk tolerance. Rather than using a flowgate constraint as a representation of transmission congestion, Fixed Transmission Rights (FTRs) have been utilized. FTRs are financial contracts that entitle the holder to a stream of revenues (or charges) based on the hourly energy price differences across the path. The model incorporates the following prices in power procurement optimization: power purchase agreements, forwards, options, and day-ahead prices. Hull (2006) provides a comprehensive review of derivative instruments. Power purchase agreements are considered the least risky way to procure power. Selling and buying agents agree on a $/MWh price for power purchased over a period of time. Some contracts last as long as twenty years and have provisions that increase the price of the contract to combat inflation. The forward market offers a contract for power whereby the seller and buyer agree on a price for an assessment period in the future.

In general, a company will want to maximize its profits by determining the optimal way to purchase power at the beginning of the cycle. In January, decisions are made about what instruments to use for power procurement for the summer peak season. In June, July and August come and the utility must use the instruments that were chosen to procure power. At the end of the year the company evaluates its decisions to determine if it has met its goals in terms of profit and risk. This problem seeks to maximize the expected profit. Revenue consists of each instrument multiplied by its respective price. The FTR auction price is then compared against the difference between the two nodes to arrive at whether the company won or lost by purchasing the FTR.

The data that are analyzed with the model were obtained from Platts' database. All data, with the exception of the Platts Megawatt Daily forward prices, are publicly available on the PJM or New York Mercantile Exchange (Nymex) websites. PJM is used for the analysis because it is a mature independent system operator with fixed transmission rights auctions and nodal pricing data. The data include PJM hourly day-ahead prices for summer 2007, PJM Financial Transmission Rights (FTR) auction prices for the assessment period of June, July and August of 2007, PJM daily forward prices for the summer months of 2007, New York Mercantile Exchange (Nymex) future prices for summer 2007, and PJM hourly load data from the summer of 2007. In order to determine which PJM nodes could be used in the model, first an analysis is performed on possible nodes with FTR data for the three summer months of 2007 that could be used as a simplistic three-node unidirectional network, which enables transmission constraint analysis. The summer of 2007 is used for the empirical implementation of the model because it was the most recent set of summer peaking season data available when this study was begun.

Valid sources and sinks for the PJM FTR auction are limited to: hubs, zones, aggregates, interface buses, load buses and generator buses. PJM hubs are reference nodes at which standard energy goods are traded. Hubs serve as a common point, or reference price, for commercial trading. The hubs are fixed weighted averages of the LMP at a set of typical buses for the chosen area. Hub prices are demonstrative of the PJM market, are fairly steady under many system conditions and are not interfered with by local transmission confines or system topology variations. Zones are a collection of load-weighted LMPs and correspond to transmission zones. Each participating electric distribution company has its own transmission zone through which it supplies its customers. An aggregate node represents a portion of the nodes that exist in the zones. The node is created at the request of the distribution utility and can be either generation- or load-weighted. Interface buses are those which connect two adjacent transmission areas. Generator buses are located adjacent to the major generating units within PJM.

The network is comprised of the nodes: Greenbri138 KV T1, Hinton 138 KV T1, and Roncever138 KV T1T3T5. These nodes are located in the American Electric Power (AEP) zone in West Virginia and belong to one of its subsidiary utilities, Appalachian Power. Greenbri138 KV T1, Hinton 138 KV T1 and Roncever138 KV T1T3T5 all represent load nodes.

Each FTR is classified as an obligation FTR, which means that the selling entity is allocated the FTR based on its load. Since the focus of the problem is from a utility's perspective, the sign on the peak prices is changed to represent what the FTR is worth to the selling agent. Table 1 shows the original data.

Table 1: FTR Obligation Prices – 2007

Source Node	Sink Node	Month	Peak Prices
GREENBRI138 KV T1	HINTON 138 KV T1	June	-512.85
		July	-290
		August	-211.41
HINTON 138 KV T1	RONCEVER138 KV T1T3T5	June	501.85
		July	282
		August	200
RONCEVER138 KV T1T3T5	HINTON 138 KV T1	June	11
		July	8
		August	11.41

The Platts Megawatt Daily forward prices are collected from a random anonymous selection of market participants. They indicate the trade date, the assessment period, whether the agreement is for peak/off peak power and the price in dollars per megawatt-hour. The nodes are managed by Appalachian Power, which is a subsidiary of AEP (American Electric Power). In order to obtain nodal level data for the forward prices, a spread between the day-ahead price for the AEP Hub and the price for each node is calculated. That spread is then multiplied with the AEP forward price in order to obtain a unique forward price for each node.

The New York Mercantile Exchange (Nymex) provides monthly futures contracts to customers based on the daily floating price for each peak day of the month at the AEP-Dayton Hub. Additional hedging opportunities are offered through options on the contracts.

Unfortunately, load data at the nodal level is not available for the PJM market. PJM only releases data at the load zone level, which only encompasses 18 entities. For this study's purpose, the hourly load data for the AEP zone are chosen, since the given nodes all reside in that zone. Per PJM data, each of these three nodes represents around 0.08 percent of the zonal data. The fact that each node represents the same fraction does not lend itself to having unique demand data for each node. Using nodal pricing data to estimate demand at each node was considered, but that the estimation would not be correct because nodal price is set not only by demand but also by other factors such as system topology, weather, demand at other surrounding nodes, etc. Therefore, the assumption is made that demand at each node could vary by ten percent in either the positive or negative direction and a random number generator was applied so that demand at each node would have the possibility of being unique. The data represent the daily peak load for days when the market was open.

In order to estimate different portfolio combinations, the load duration curve for AEP for the summer of 2007 is utilized. A load duration curve demonstrates a company's load in megawatts from largest to smallest load for a given time.

As a general rule, a utility may serve around 85 percent of its peak by more secure contracts like power purchase agreements; the remaining 15 percent would be obtained through forwards, options and the spot market (based on conversation with industry experts). When the company makes procurement decisions, it does not know what the actual load will be. Therefore, it may not need to use the spot market because the load may not be high enough to warrant additional power purchases. It follows that two of the scenarios do not contain any spot purchases. Forward data are available for power bought up to 12 months before power delivery, but only the forwards that are purchased up to five months were used; it is assumed that a utility begins to think about summer peak season in January of the same year. In total eight scenarios are considered.

The probability of each scenario being chosen is generated by Monte Carlo simulation (using an add-in tool called YASAI Simulation Version 2.0 in Microsoft Excel) for each of the three nodes, which gives the average profit for each scenario. The profit differential is then used to estimate the probability that each scenario will be used by the utility to purchase power.

In order to calculate VaR and CVaR values, the Monte Carlo simulation method is used. Combinations of the eight scenarios for each node are evaluated for a total of 512 scenarios. The same random number seed is used for each scenario and each run has a sample size of 1,000. The simulation with an output of 512 scenarios is run a total of ten times because there is very little variation in output in each of the ten runs. The slight numerical difference among the ten simulation runs is probably due to the fact that historical data are used for the analysis. The variation in pricing and demand data that is normally observed is eliminated by using actual data from 2007. The estimated nodal demand and actual nodal prices are used to determine the profit for each of the scenarios. The probability for the combination of scenarios is calculated from the combined probabilities of each of the component scenarios. Figure six displays the expected profit distribution.

The confidence level for the problem is set at 95 percent based on the level most-used by previous studies in the area of power portfolio optimization. The scenarios are then sorted in ascending order based on expected profit. In order to find the VaR, the probabilities of the scenarios (starting with the most negative profit) are added up until the five percent VaR is reached. The value is found to lie between the 88th and 89th records and is equal to a profit of around $1,216. The CVaR is then calculated basically as the weighted average of the tail from the most negative profit to the VaR profit value. This method is used in order to take into account possible extreme behavior in the tail as shown in Sarykalin et al (2008). The value for the CVaR is calculated at a loss of $82. The fact that the CVaR is negative signifies that the VaR does not take into account extreme losses in the tail. Given a negative CVaR, the utility may want to decrease the confidence level to 90 percent. Table 2 is a snapshot of the Monte Carlo Simulation analysis.

Table 2: Monte Carlo Simulation for VaR/CVaR Calculation

Record	Scenario	Probability	Observations	Mean Profit
1	512	0.0156%	1000	$ (5,800.47)
2	448	0.0156%	1000	$ (5,111.05)
3	511	0.0313%	1000	$ (5,025.84)
4	504	0.0156%	1000	$ (5,025.11)
5	447	0.0313%	1000	$ (4,336.41)
⋮	⋮	⋮	⋮	⋮
84	310	0.0469%	1000	$ 1,187.08
85	380	0.1250%	1000	$ 1,188.34
86	303	0.0313%	1000	$ 1,199.12
87	407	0.0313%	1000	$ 1,199.75
88	352	0.0156%	1000	**$ 1,214.14**
89	442	0.3125%	1000	**$ 1,260.75**

RESULTS AND SENSITIVITY ANALYSIS

The nonlinear stochastic mixed-integer model is run using the Solver tool in Microsoft Excel. Table 3 shows the vital statistics from the results.

Table 3: Results from Optimization

GREENBRI138 node (Scenario = Conservative (plan early))							
%PPA	**%FOR1**	**%FOR2**	**%FOR3**	**%FOR4**	**%FOR5**	**%OPTION**	**%SPOT**
88%	2%	0%	0%	1%	4%	5%	0%
Qppa	**Qfor1**	**Qfor2**	**Qfor3**	**Qfor4**	**Qfor5**	**Qcall**	**Qspot**
1,019.87	23.18	0.00	0.00	11.59	46.36	57.95	0.00
Profit				**Added Qspot for transmission constraint**			
$ 8,983.75				0			
RONCEVER138 node (Scenario = Conservative (plan early))							
%PPA	**%FOR1**	**%FOR2**	**%FOR3**	**%FOR4**	**%FOR5**	**%OPTION**	**%SPOT**
88%	2%	0%	0%	1%	4%	5%	0%
Qppa	**Qfor1**	**Qfor2**	**Qfor3**	**Qfor4**	**Qfor5**	**Qcall**	**Qspot**
1,021.24	23.21	0.00	0.00	11.61	46.42	58.03	0.00
Profit				**Added Qspot for transmission constraint**			
$ 10,407.71				0			
HINTON node (Scenario = Conservative (plan early))							
%PPA	**%FOR1**	**%FOR2**	**%FOR3**	**%FOR4**	**%FOR5**	**%OPTION**	**%SPOT**
88%	2%	0%	0%	1%	4%	5%	0%
Qppa	**Qfor1**	**Qfor2**	**Qfor3**	**Qfor4**	**Qfor5**	**Qcall**	**Qspot**
1,016.19	23.10	0.00	0.00	11.55	46.19	57.74	0.00
Profit				**Added Qspot for transmission constraint**			
$ (5,465.44)				5.75			
TOTAL PROFIT							
$ 13,926.02							

The results show that all three nodes arrive at an optimal solution with the same strategy – the one in which the utility has a fairly high percentage of its power coming from fixed-price power contracts and purchases the majority of the rest of its needed power in the five-month forward market and in options. Rather than using the spot market, the utility estimates the max peak for the following month and purchases one-month forward contracts for that amount. The only spot purchased is for the extra demand at the Hinton node in order to satisfy the transmission constraint. The Hinton node generally has a negative profit because of the unfavorable conditions of the financial transmission rights contract from the Greenbri138 node to the Hinton node and because of the additional demand at Hinton that must be satisfied through the spot market. One assumes the utility will reevaluate its positions at this node for the following summer season so that the profit will have the opportunity to turn positive. A total profit of $13,926 is reasonable because it represents only the earnings from three nodes in a total network of hundreds of nodes.

Since each of the scenarios has a fairly high percentage of its supply coming from long term power purchase agreements (the lowest percentage being 64), the price chosen for the fixed contract will have a large effect on the outcome. The power purchase price for the optimization is set at $64 per MW based on a sampling of released prices to various regulatory agencies by utilities. By increasing this price by only two dollars, the profits are cut in half. Once the contract price is increased to $69, all profits are negative. In addition, some of the scenarios with a lower percentage of power purchased through long term contracts begin to appear in the top ten scenario combinations' profits. For the $66 trial, the "middle (plan early)" scenario (number four) is chosen for the Hinton node for the fifth highest profit, whereas in the initial optimization, this scenario did not appear until the eighth highest profit. For the $69 trial, it rises to the scenario combination with the second-highest profit. The results displayed in Table 4 emphasize just how important the power purchase agreement price is for the analysis.

Table 4: Power Purchase Agreement Analysis

Base Case (a = $64/MW)				Middle Case (a = $66/MW)				High Case (a = $69/MW)			
Highest Profits				**Highest Profits**				**Highest Profits**			
Profit	No.G	No.R	No.H	Profit	No.G	No.R	No.H	Profit	No.G	No.R	No.H
$13,926.02	7	7	7	$7,811.41	7	7	7	($1,360.50)	7	7	7
$13,471.86	8	7	7	$7,264.54	8	7	7	($1,740.41)	7	7	4
$13,464.63	7	8	7	$7,257.18	7	8	7	($1,992.82)	7	4	7
$13,462.29	7	7	8	$7,255.31	7	7	8	($1,995.31)	4	7	7
$13,010.47	8	8	7	$7,015.79	7	7	4	($2,046.44)	8	7	7
$13,008.14	8	7	8	$6,761.30	7	4	7	($2,053.99)	7	8	7
$13,000.90	7	8	8	$6,759.38	4	7	7	($2,055.17)	7	7	8
$12,853.26	7	7	4	$6,710.31	8	8	7	($2,372.73)	7	4	4
$12,597.39	7	4	7	$6,708.43	8	7	8	($2,375.22)	4	7	4
$12,595.84	4	7	7	$6,701.07	7	8	8	($2,426.35)	8	7	4
Lowest Profits				**Lowest Profits**				**Lowest Profits**			
Profit	No.G	No.R	No.H	Profit	No.G	No.R	No.H	Profit	No.G	No.R	No.H
($5,800.47)	2	2	2	($10,525.39)	2	2	2	($17,612.78)	2	2	2
($5,111.05)	2	2	1	($9,743.59)	2	2	1	($16,692.40)	2	2	1
($5,025.84)	1	2	2	($9,658.04)	1	2	2	($16,606.36)	1	2	2
($5,025.11)	2	1	2	($9,657.20)	2	1	2	($16,605.32)	2	1	2
($4,336.41)	1	2	1	($8,876.24)	1	2	1	($15,685.98)	1	2	1
($4,335.69)	2	1	1	($8,875.39)	2	1	1	($15,684.95)	2	1	1
($4,250.48)	1	1	2	($8,789.85)	1	1	2	($15,612.84)	2	2	5
($3,561.06)	1	1	1	($8,109.74)	2	2	5	($15,598.90)	1	1	2
($3,107.67)	2	2	5	($8,008.04)	1	1	1	($15,216.95)	5	2	2
($2,709.27)	5	2	2	($7,712.34)	5	2	2	($15,205.12)	2	5	2

Although the model results depend on the options pricing, the prices were not available at the nodal level from Nymex. For the forward prices, the lack of nodal-level pricing was solved by taking the spread between the nodal and zonal spot prices and applying the percent difference to the zonal forward prices in order to arrive at a nodal pricing scheme. If the same methodology had been applied to the options, there would have been a direct correlation between the options and forward prices for each node. Since the prices are then not evaluated at a nodal level, the question arises whether or not they should be included in the model. Table 5 demonstrates the comparison between the results with and without options.

Table 5: Results with and without Options

With Options				Without Options			
Highest Profits				**Highest Profits**			
Profit	No.G	No.R	No.H	Profit	No.G	No.R	No.H
$13,926.02	7	7	7	$11,950.82	7	7	7
$13,471.86	8	7	7	$11,941.13	8	7	7
$13,464.63	7	8	7	$11,931.30	7	8	7
$13,462.29	7	7	8	$11,921.61	8	8	7
$13,010.47	8	8	7	$11,833.72	7	7	8
$13,008.14	8	7	8	$11,824.03	8	7	8
$13,000.90	7	8	8	$11,814.20	7	8	8
$12,853.26	7	7	4	$11,804.51	8	8	8
$12,597.39	7	4	7	$10,324.30	7	7	4
$12,595.84	4	7	7	$10,314.61	8	7	4
Lowest Profits				**Lowest Profits**			
Profit	No.G	No.R	No.H	Profit	No.G	No.R	No.H
($5,800.47)	2	2	2	($9,645.26)	1	1	1
($5,111.05)	2	2	1	($9,523.01)	1	1	2
($5,025.84)	1	2	2	($9,413.95)	1	2	1
($5,025.11)	2	1	2	($9,407.58)	2	1	1
($4,336.41)	1	2	1	($9,291.69)	1	2	2
($4,335.69)	2	1	1	($9,285.33)	2	1	2
($4,250.48)	1	1	2	($9,176.26)	2	2	1
($3,561.06)	1	1	1	($9,054.01)	2	2	2
($3,107.67)	2	2	5	($6,484.63)	1	1	5
($2,709.27)	5	2	2	($6,363.08)	1	1	3

The profits when the model does not include options are generally around ten to 20 percent lower than when options are included. The optimal scenario combination (7, 7, 7) does not change, but the lowest profit scenario combination changes from each node choosing the “low base (plan late)” scenario to choosing the “lowest base” scenario. This result shows that the utility is putting even more emphasis on the power purchase agreement price since the “lowest base” scenario has the lowest percent of power being obtained through long-term contracts. In addition, the VaR plummets into the negatives to -$1,122. This value is around a 200 percent drop from the original positive $1,216. The lower profits can be justified by looking at the Nymex prices. In general, they are fairly comparable to the forward prices. When they are dropped from the model, the old percentage of options purchased is spread out among other instruments, whose prices are generally higher than the options prices. The profit suffers from subtracting one of the choices. One could also hypothesize that by subtracting a decision variable from the model, the robustness of the objective function suffers.

The main role of this study is the addition of transmission-related constraints to the power portfolio optimization problem. This contribution is satisfied in two main ways: a transmission constraint to make a unidirectional three-node system and fixed transmission rights that are included in the objective function. The question remains if the constraint and FTRs affect the overall solution and whether they are worthwhile. The model is run three additional times: once with no transmission constraint, once with no FTRs and once with neither. Table 6 contains the varying ten highest and ten lowest profits for each run along with the VaR and CVar values.

Table 6: Results of Model with Varying Transmission Considerations

With FTR & Constraint	Without FTR, With Constraint	With FTR, Without Constraint	Without FTR & Constraint
	Highest Profits		
$13,926.02	$25,033.92	$14,967.60	$26,075.49
$13,471.86	$24,579.76	$14,513.44	$25,621.34
$13,464.63	$24,572.53	$14,506.20	$25,614.10
$13,462.29	$24,570.19	$14,503.87	$25,611.77
$13,010.47	$24,118.37	$14,052.05	$25,159.94
$13,008.14	$24,116.03	$14,049.71	$25,157.61
$13,000.90	$24,108.80	$14,042.48	$25,150.37
$12,853.26	$23,961.15	$13,894.83	$25,002.73
$12,597.39	$23,705.29	$13,638.97	$24,746.86
$12,595.84	$23,703.73	$13,637.41	$24,745.31
	Lowest Profits		
($5,800.47)	$5,307.43	($4,758.89)	$6,349.00
($5,111.05)	$5,996.85	($4,069.47)	$7,038.42
($5,025.84)	$6,082.06	($3,984.26)	$7,123.64
($5,025.11)	$6,082.78	($3,983.54)	$7,124.36
($4,336.41)	$6,771.48	($3,294.84)	$7,813.06
($4,335.69)	$6,772.21	($3,294.11)	$7,813.78
($4,250.48)	$6,857.42	($3,208.90)	$7,898.99
($3,561.06)	$7,546.84	($2,519.48)	$8,588.42
($3,107.67)	$8,000.22	($2,066.10)	$9,041.80
($2,709.27)	$8,398.63	($1,667.70)	$9,440.20
	VaR		
$1,215.72	$12,337.50	$2,258.59	$13,380.38

	CVaR		
($82.05)	$10,174.84	$959.52	$12,067.42

The results suggest that the transmission considerations in the model do make a difference in the overall optimization. It makes sense that the inclusion of transmission in a power portfolio optimization would be important. A utility not only has to take into consideration what its customers' needs will be in the future but also how to optimally deliver the power to its customers. There is not only risk involved with the financial instruments chosen but also with the uncertainty of transmission conditions. The effect of the FTR on the Hinton node suggests that the utility would need to reconsider its strategy to hedge congestion on that particular line.

It is not surprising that the FTR effect is more potent than the transmission constraint's effect. Unfortunately, the model does not contain nodal demand data and each node seems to represent the same percent of the AEP zone. Since the nodal demand data are estimated from the percent each node represents of the zone times a random number generator, the transmission constraint does not represent the actual system. The real constraint could vary greatly from the one presented in this paper. Overall, though, the presence of the constraint goes to prove the importance of its existence even if it is not quantified correctly.

CONCLUSION

Many power portfolio optimization problems have been developed to combat the issue of risk tolerance, but very few (if any) have included transmission constraints. In this research, optimization of portfolios of real and contractual assets, including derivative instruments, in a multi-period setting is considered where transmission constraints also exist. Fixed transmission rights are used as a measure of transmission congestion using data from the PJM market, which is located in the eastern United States. PJM, the most mature independent system operator, was used for model implementation because it uses a nodal pricing system (rather than a zonal average) and data are readily available from PJM's website.

The results presented in this paper show that transmission considerations in a power portfolio optimization problem do have an impact on the profit function. By omitting transmission congestion from their models, previous studies may have over-stated expected profit. Transmission constraints are a type of unknown risk in the power sector; by mitigating the risk, power companies will have to accept lower profits; however, the probability of sustaining extreme losses would be biased if these constraints are not considered. While the model presented in this paper does not find that companies will change their specific procurement decisions when a transmission constraint is introduced, further research with company-specific data could provide more conclusive results.

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